

Quantum Gravity, Cosmology, (Liouville) Strings and Lorentz Invariance

Nick E. Mavromatos

King's College London, Department of Physics, Theoretical Physics, Strand, London WC2R 2LS, U.K.

Abstract. I review some aspects of non-critical strings in connection with Lorentz-Invariance violating approaches to quantum gravity. I also argue how non-critical strings may provide a unifying framework where string Cosmology and quantum gravity may be tackled together.

1. Introduction

Recently there have been exciting developments in astrophysics and cosmology, associated with observations of ultra high energy cosmic rays (UHECR), of energies larger than 10^{20} eV [1], as well as high-redshift ($z \sim 1$) supernovae observations [2] claiming a current-era acceleration of the Universe. The latter observations, when combined with data from measurements of the cosmic microwave background radiation [3], which provide evidence for a spatially flat universe, with a $\Omega_{\text{total}} = 1$, seem to indicate that up to 70 % of the total energy density of the Universe is attributed to a *dark energy* component, which could be a positive cosmological constant (de Sitter type Universe) Λ or something else. On the other hand, the presence of UHECR events appears puzzling from the point of view of Lorentz Invariance [4, 5, 6]. The latter symmetry would impose an energy threshold (Greitsen-Zatsepin-Kuzmin (GZK) cut-off) [7] above which certain reactions can occur which would prevent such UHECR events to reach the observation point, if one makes the physically plausible assumption that such events come from extragalactic sources [1].

These data present serious puzzles and challenges for the quantum field theory as we know it. First, if there is a non-zero cosmological *constant* Λ , (de Sitter phase in the Universe), this would eventually mean that the (constant) vacuum energy will dominate over matter in the Universe, given the a^{-3} relaxation of the latter with the expanding-universe scale factor $a(t)$. When the vacuum energy is dominant the Universe will enter again a de Sitter inflationary phase, with exponential growth, $a(t) \sim a_0 e^{\sqrt{\frac{\Lambda}{8\pi G_N}} t}$ (G_N is the gravitational (Newton's) constant). With such scale factors, there will be cosmic horizons, since a light ray in a Robertson-Walker type Universe will take an infinite time to traverse a *finite distance*

$$\delta \sim \int_{t_0}^{\infty} \frac{cdt}{a(t)} < \infty, \quad \text{for } a(t) \sim a_0 e^{\sqrt{\frac{\Lambda}{8\pi G_N}} t} \quad (1)$$

The presence of horizons imply the impossibility of defining pure asymptotic states, which are essential for the proper definition of a Heisenberg scattering S -matrix, and thus a conventional quantum field theory in such gravitational backgrounds. This is a

serious challenge also for string theory [8], which, by its very nature, is supposed to be a theory of S-matrix.

Second, a relaxation of the GZK cutoff -in order to explain the UHECR events- would, in turn, imply a relaxation of Lorentz invariance [4, 5], which is another drastic modification of field theory. Along these lines it has also been suggested [9] that deviation from Lorentz symmetry, in the sense of modified dispersion relations for matter probes, can also explain the presence of TeV cosmic photon events, which again should have been prevented from reaching the observation point within the standard kinematics stemming from Lorentz invariance requirement during the scattering of energetic photons with the infrared background radiation.

A question I would like to address in this talk is whether there is a unified framework in which the above puzzles can be tackled, with the simultaneous ability to make concrete, experimentally falsifiable, predictions. In addition, I will attempt to take one more challenge, and tackle, within the above framework, another important issue, that of the *hierarchy* between the currently claimed [2, 3] “observed value” of the “vacuum energy”, $\Lambda/M_P^4 < 10^{-123}$ in Planck M_P units, and the *supersymmetry breaking scale*, of a few TeV, in supersymmetric theories.

For the purposes of my talk I will concentrate in the string theory framework, and present some toy models of (non-critical) strings [10] where the above issues can be tackled, as I will argue, in a mathematically consistent way. The structure of the talk is the following: in section 2 I present a very brief overview of possible violations of Lorentz Invariance, and the associated consequences, from various theoretical viewpoints. In section 3 I discuss the issue of Lorentz Invariance in string theory. I start from critical string theory, where Lorentz invariance is valid, and then proceed to explain carefully how non-critical string models may lead to its violation, in the sense of modified dispersion relations for matter probes (refractive indices *etc*). In section 4 I describe briefly two concrete models with the above properties: (i) one model for space time foam in Liouville strings, involving the interaction of string matter with stringy space-time defects, playing the rôle of space time “foam”, and (ii) a non-critical stringy model of Cosmology, where the non-criticality, which is here viewed as departure from equilibrium, is provided by an initial catastrophic event, playing the rôle of Big Bang. The model consists of two colliding branes. In the context of this model I will discuss relaxation scenaria for the cosmological vacuum energy, inflation and graceful exit from it, and how an S-matrix can be ultimately salvaged by exiting from the current-era accelerating phase. The model involves two time-like variables, one of which is the Liouville mode, required for consistency of the non-critical string.

2. Lorentz Symmetry Violations and Quantum Gravity: A Brief Overview of Models and Approaches

I adopt the point of view that Lorentz symmetry is a good symmetry of any isolated quantum field theory model on flat space time, and that its possible violations come from either placing the system in a ‘medium’ or heat bath (finite temperature), or coupling it to (certain models of) quantum gravity, which also behave like a stochastic medium [10, 11].

A theory of quantum gravity must describe Physics at Planck scales, $\ell_p \sim 10^{-35}$ m. At such small length scales the structure of space time might be very different from what we have experienced so far, *e.g.* the space time might be discrete, non commutative *etc*. Therefore Lorentz symmetry might not be valid at such small scales, or at least its form may be different from the familiar one characterising the relatively low energy scales

of the current experiments (for instance, the Lorentz symmetry might be non-linearly realized at Planckian scales).

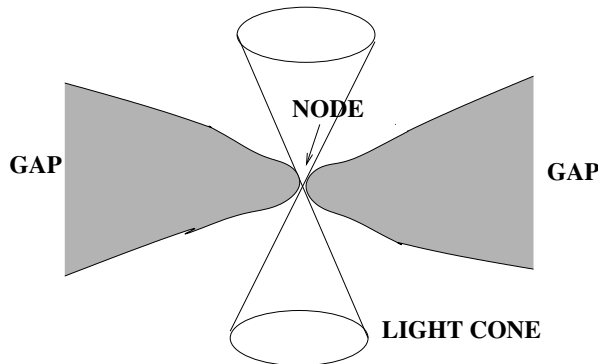


Figure 1. A condensed matter system with nodes in its Fermi surface (or energy gap). Linearizing the quasiparticle-excitation effective theory around the nodes one obtains relativistic field theories in the continuum limit. The ‘induced light cone’ has a limiting velocity which is given by the fermi velocity of the node. Such systems provide analogues of models where Lorentz Symmetry is a good symmetry only at low energies.

There are instructive examples from condensed matter physics, that probably help understand physically a possible breakdown of Lorentz symmetry at higher energies. For instance, Antiferromagnetic Spin Systems at low energies are known to be described effectively by *relativistic* continuum field theories, for instance the large spin $S \rightarrow \infty$ limit of planar antiferromagnetic lattice spin systems yields a relativistic CP^1 σ -model with action $\mathcal{S} = \int d^3x \frac{1}{\gamma_0} |(\partial_\mu - a_\mu)z|^2$, $\gamma_0 \propto 1/S$, $S \rightarrow \infty$, z are magnons (spin degrees of freedom), and the non-dynamical gauge field a_μ takes proper account of the correct number of physical degrees of freedom in the system. Another instructive example is that of systems with fermionic quasiparticle excitations which have *nodes* in their fermi surface (or energy gap) (c.f. figure 1). The (average) radius of the fermi surface provides the energy reference scale above which all energies of quasiparticle excitations are measured. Low energies in this setting are therefore viewed those near the fermi surface. Near the node, where the gap function vanishes $\Delta \rightarrow 0$, the dispersion relation of the quasiparticle excitation reads: $E = \sqrt{\frac{|\vec{k}|^2}{2\mu} + \Delta^2} \simeq \frac{1}{\sqrt{2\mu}} |\vec{k}|$, with μ the Fermi velocity of the node, playing here the rôle of a limiting velocity (‘speed of light’) for the relativistic problem at hand. Linearizing the fermionic quasiparticle spectrum about the node one may therefore obtain a relativistic low-energy field theory in the continuum limit consisting of Dirac fermions: $\mathcal{S}_F = \int d^Dx \bar{\Psi}(\gamma^\mu \partial_\mu + \dots)\Psi$, $\mu = 0, 1, 2, \dots$. Such a condensed-matter inspired approach has been adopted in the literature in an attempt to discuss the origin of Lorentz symmetry in quantum field theory [12].

In the physics of fundamental interactions deviations from Lorentz symmetry may become manifest in modifications of the dispersion relations of matter probes. For massless probes such modifications imply non-trivial refractive indices due to the modification in the group velocity of the probe. For instance, in theories with non-trivial vacuum polarization at finite temperature, such as quantum electrodynamics *etc.*, one obtains non-trivial refractive indices already at one loop [13], in the sense of a modified

dispersion

$$\frac{\partial E}{\partial |\vec{p}|} = 1 + f(T, |\vec{p}|) , \quad (2)$$

with T the temperature, and the function f being determined from the vacuum polarization graph.

It is this sort of modification that may be induced also by quantum gravity effects. Indeed, one of the most fascinating ideas about quantum gravity is the suggestion made by John Wheeler, and subsequently adopted by Stephen Hawking, that space time at Planckian scales might acquire a foamy structure (c.f. figure 2), which may thus behave like a stochastic ‘medium’, with non-trivial macroscopic consequences for the propagation of matter probes in such backgrounds [10, 14, 11]. This results in modified dispersion relation and other non-trivial “optical” properties.

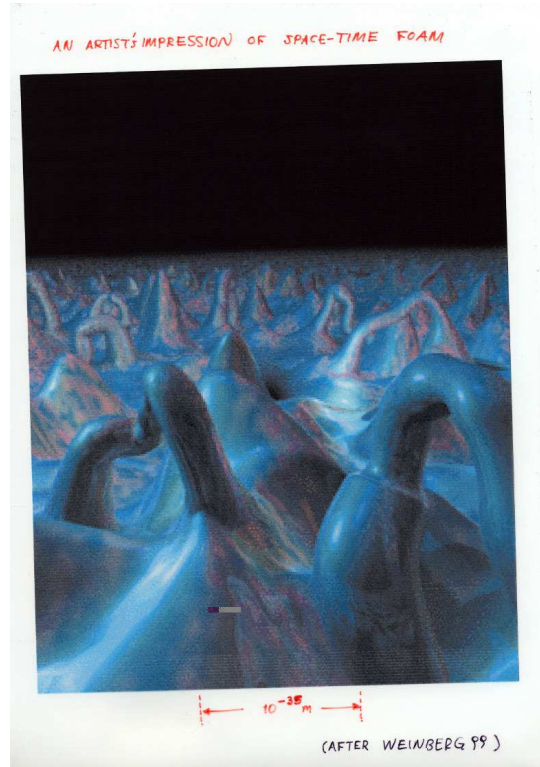


Figure 2. An artist impression of space-time foam (after S. Weinberg 1999; this figure was kindly provided to the author by Subir Sarkar).

Such modified dispersion relations are known to characterise non-critical (Liouville) string theory models of space-time foam [10], where the departure from criticality (conformality of the associated σ -model background) is viewed here as a consequence of the interaction of string matter with the (quantum gravity) foam:

$$E^2 = |\vec{p}|^2 + F(E, |\vec{p}|; \ell_p) , \quad (3)$$

where the function F is suppressed by higher powers of the quantum-gravity scale (Planck (or string) length M_P (M_s)). Subsequently, similar phenomena have been argued [15] to characterise the so-called loop gravity approach [16].

The latter is a background independent theory of gravity, in which the space time is characterised by a ‘polymer-like’ discrete structure. As a result of this property, the

basic state of the theory, the “weavy state”, $|w\rangle$ is such that, although macroscopically the (emergent) space time looks Minkowski, nevertheless there is non-trivial structure at scales smaller than the characteristic scale ℓ_W where quantum gravity effects set in: $\langle w|G_{\mu\nu}|w\rangle = \eta_{\mu\nu} + \mathcal{O}(\ell_w E)$, where E is the energy of the probe. Such effects lead to non-trivial dispersion relations, similar in nature to those predicted in non-critical strings [10]. There is an important difference, though, which distinguishes experimentally strings from these theories: in loop gravity superluminal propagation of matter probes is allowed, implying birefringence effects. Such superluminal signals are forbidden in the Liouville (non-critical) string approach to quantum gravity, for specifically stringy reasons [10].

There is an important difference of the modification of the dispersion relation induced by quantum gravity, as compared to conventional vacuum polarization effects [13], in that the former increases with the energy of the probe, in contrast to the conventional field theoretic effects which decrease with energy. This allows a possible disentanglement of such effects in experiments. One of the most sensitive probes of such non-trivial optical properties is the observation of light from Gamma Ray Bursters (GRB) [14] or gamma ray flairs [17]. As mentioned in the introduction, such modified dispersion relations, have been used subsequently to explain the transparency of the Universe in ultra high energy cosmic rays [5] or TeV photon events [9]. For a recent and comprehensive review of astrophysical tests of such quantum-gravity-induced modified dispersion relations see [6].

Finally, modified dispersion relations for matter probes have been recently [18, 19] argued to characterise flat space theories in certain models in which the Planck “length” is considered a real length. As such it should be transformed under usual Lorentz boosts. The requirement that the Planck length be, along with the speed of light, also *observer independent* leads quite naturally to modified “Lorentz transformations”, and also to modified dispersion relations for particles. This approach is termed “Doubly Special Relativity” (DSR) [18]. The DSR approach should be distinguished from the other approaches, mentioned so far, where the Planck length scale is associated to a ‘coupling constant’ of the theory, and thus is observer independent by definition. This is clearly the case of Einstein’s General Relativity, where the Planck scale is related to the universal gravitational coupling constant (Newton’s constant), and in string theory (critical and non-critical), where the Planck length is related to the characteristic string scale of the theory, which is also independent of inertial observers.

There are two approaches so far in the problem of formulating DSR theories: in the first [18] one postulates a modified dispersion $E^2 = |\vec{p}|^2 + f_1(E, |\vec{p}|; \tilde{\ell}_P)$, where $\tilde{\ell}_P \propto \ell_P$ is the inertial-observer independent length, proportional to the conventional Planck length $\ell_P \sim 10^{-35}$ m. The approach is as yet at a phenomenological level, and hence one can choose appropriate functions f_1 . Let me illustrate the main features with one such choice: $f_1 = -\tilde{\ell}_P^{-2} (e^{\tilde{\ell}_P E} + e^{-\tilde{\ell}_P E} - 2) + |\vec{p}|^2 e^{\tilde{\ell}_P E}$. The generators of the deformed Lorentz boosts (assumed, for definiteness and simplicity, to be along z direction), that leave $\tilde{\ell}_P$ observer independent, read [18]:

$$\mathcal{N}_z = p_z \frac{\partial}{\partial E} + \left(\frac{\tilde{\ell}_P |\vec{p}|^2}{2} + \frac{1 - e^{-2\tilde{\ell}_P E}}{2\tilde{\ell}_P} \right) \frac{\partial}{\partial p_z} - \tilde{\ell}_P p_z \left(p_j \frac{\partial}{\partial p_j} \right) \quad (4)$$

As mentioned previously an important consequence of the formalism is that by construction there are modified dispersion relations:

$$m^2 = \tilde{\ell}_P^{-2} (e^{\tilde{\ell}_P E} + e^{-\tilde{\ell}_P E} - 2) - |\vec{p}|^2 e^{\tilde{\ell}_P E} , \quad (5)$$

where m is the rest mass of a matter point-like probe, defined by the appropriate quadratic Casimir operator. Note that m is different from the *rest energy* \mathcal{M} , the latter defined as the energy at zero momentum $\vec{p} = 0$:

$$m = \tilde{\ell}_P^{-1} \left(e^{\tilde{\ell}_P \mathcal{M}/2} - e^{-\tilde{\ell}_P \mathcal{M}/2} \right) \quad (6)$$

In this particular choice of f_1 note that the rest mass differs from the rest energy only by terms of order $\tilde{\ell}_P^2$ and higher: $m = \mathcal{M} + \mathcal{O}(\tilde{\ell}_P^2)$.

It was remarked in [18] that the modified dispersion implies a group speed of photon $v_\gamma = \frac{\partial E}{\partial p} = e^{\tilde{\ell}_P E} = 1 + \tilde{\ell}_P E + \dots$ which diverges as $E \rightarrow \infty$. However I note here that the meaning of $E \rightarrow \infty$ is not clear, especially in a theory with gravity, where in principle one expects the whole structure of space time to be changed drastically at energies close to Planck scale, which thus acts as an effective ultraviolet cutoff.

In the second approach [19], one combines Lorentz boosts with dilatations in order to achieve the observer-independence of the Planck length (we assume again the boost along z -direction for comparison with the previous approach):

$$\mathcal{N}'_z = \left[p_z \frac{\partial}{\partial E} \right] + E \frac{\partial}{\partial p_z} - \tilde{\ell}_P p_z \left(E \frac{\partial}{\partial E} + p_j \frac{\partial}{\partial p_j} \right) \quad (7)$$

where the first term inside square brackets denotes the boost L_0^z , and the rest the dilatation \mathcal{D} . The combined dilation and boost algebra closes with the three-rotations:

$$J^i = \epsilon^{ijk} L_{jk}, K^i \equiv L_0^i + \tilde{\ell}_P p^i \mathcal{D}, [J^i, K^j] = \epsilon^{ijk} K_k, [K^i, K^j] = \epsilon^{ijk} J_k.$$

There is a modified dispersion relation in this approach [19]:

$$m^2 = \frac{E^2 - |\vec{p}|^2}{(1 - \tilde{\ell}_P E)^2} = \frac{\eta^{\mu\nu} p_\mu p_\nu}{(1 - \tilde{\ell}_P E)^2}. \quad (8)$$

Notice that in this second approach, in contrast to that of [18], all energies are necessarily smaller than the ‘gravity scale’, $E < \tilde{\ell}_P^{-1}$, given that at such scales there appears to be a ‘metric’ collapse in the dispersion relation. This approach is thus closer to the conventional understanding of the Planck scale as an ultraviolet cutoff for any low-energy theory. Another important feature is that the speed of light or, in general, of massless particles is $c(=1)$, as in Special Relativity of Einstein.

Again, the rest mass m is different from the rest energy \mathcal{M} , but here the difference already occurs already at linear order in $\tilde{\ell}_P$: $m = \frac{\mathcal{M}}{1 - \tilde{\ell}_P \mathcal{M}} \simeq \mathcal{M} + \tilde{\ell}_P \mathcal{M}^2 + \dots$. It should be noted that there are experimentally testable differences between the DSR models of [18] and [19], e.g. in connection with kinematical conditions for particle production in collisions [18].

It remains to be seen what consequences these modified flat space transformations would have in a general relativistic framework. For instance, from (8) one would be tempted to identify such effects with those in a specific curved space-time background, which as I mentioned above, seems to collapse at Planck energies. Are, then, such DSR models, when placed in a general relativistic context, selecting special metric backgrounds as, for instance, is the case of the modified dispersion relations obtained in non-critical string inspired models [10]? These and other questions, related to a reconciliation of DSR (viewed as theories with a minimum length) with quantum mechanics, should be pursued in future research.

At this point I end this brief overview of possible theoretical models predicting violations or modifications of the Lorentz symmetry. In what follows I will concentrate on specific models of such violations stemming from (non-critical (Liouville)) String Theory [10].

3. String Theory and Lorentz Invariance

3.1. Standard Dispersion Relations in Critical Strings

Let me start from “old” string theory [20], living in a critical dimension of target space time. From a first quantization viewpoint the motion of strings is described by a σ -model, a two-dimensional world-sheet field theory, in background target-space fields. There is a basic symmetry, called conformal symmetry, which allows a consistent path integral formulation of the σ -model, and restricts the background fields to their target-space classical equations of motion, thereby providing an important link between consistent world-sheet quantum geometry and target space dynamics. It is this symmetry that implies Lorentz invariance in critical number of dimensions for flat space times, and the standard dispersion relations of stringy excitations. Let us briefly see how.

Consider a bosonic σ -model in flat space time, perturbed by certain tachyon excitations (although tachyons are viewed as instabilities of bosonic strings, and are absent in superstrings, nevertheless we consider them here for simplicity and because they capture all the essential features we wish to discuss, which can be immediately generalized to the superstring case):

$$S_\sigma = \int_\Sigma \partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \frac{1}{4\pi\alpha'} \int d^D k \tilde{T}(k) \int_\Sigma e^{ik_\mu X^\mu} \quad (9)$$

where k^μ is a D -dimensional target space momentum, Σ denotes the world sheet, $\partial, \bar{\partial}$ are world-sheet derivatives, and $\alpha' = \ell_s^2$ is the Regge slope [20], related to the square of the characteristic length scale of the string ℓ_s , which is observer independent from a target-space viewpoint. In old string theory this scale was identified with the four dimensional Planck mass scale, although in the modern membrane (D-brane) extension the two scales may not be the same [21],

The exponential vertex operator $V_k = e^{ik_\mu X^\mu}$, which described the tachyonic excitation of the bosonic string spectrum, with polarization “tensor” $\tilde{T}(k)$, is not in general a marginal operator in a world-sheet renormalization group sense, and hence under the two-dimensional quantum field theory corrections will break the conformal invariance in the sense that its conformal dimension Δ is different from one:

$$\Delta = \frac{\alpha'}{2} k_\mu k^\mu \quad (10)$$

Insisting on conformal invariance, i.e. marginality of the operator, one obtains the standard dispersion relation (*on-shell condition*) for the tachyonic excitation compatible with Lorentz invariance in target space:

$$\Delta = 1 \rightarrow k_\mu k^\mu = \frac{2}{\alpha'} \quad (11)$$

with the tachyon rest mass $m^2 = -\frac{2}{\alpha'}$ (the negative sign indicates the abovementioned tachyonic instability of the bosonic string, but this does not affect the arguments on dispersion relation because the same procedure applies to all other string excitations).

The number of target-space dimensions D is also restricted by the same requirement of conformal world-sheet invariance if one performs the path integral computation of the trace of the world-sheet stress tensor $\Theta = T_{\alpha\beta} \gamma^{\alpha\beta}$ in curved world sheets. Even for a free string, then, one finds that $\langle \Theta \rangle = (D - 26)R^{(2)}$, where $\langle \dots \rangle$ denotes a two-dimensional path integral average, $R^{(2)}$ is the world-sheet curvature, and the -26 appeared due to the fact that general coordinate diffeomorphism invariance in target

space acts from a two-dimensional viewpoint as a gauge symmetry which needs fixing, thereby implying Fadeev-Popov ghosts. Their contribution to the conformal anomaly is then given by $-26R^{(2)}$. To ensure conformal invariance one must have $\langle \Theta \rangle = 0$ which select the number of space time dimensions for the bosonic string to 26 (for the superstring this number reduces to 10).

3.2. Non-critical (Liouville) Strings: a brief overview of the formalism

Critical string theory has proven a very successful and elegant formalism so far to understand many questions related to the structure of space time at Planckian distances, to count microstates in singular space times such as black holes *etc.* However, there are some situations which critical strings appear inadequate to describe. These include formation of black holes and other dynamical space-time boundaries, stochastic space-time foam backgrounds (if one adopts the point of view that the latter exist), recoil effects of stringy solitons, and in general situations which involve the change of a background over which the string propagates. Critical strings, and conventional conformal field theories, describe *fixed background* situations. Since, however, strings are supposed to be a theory of space time itself, they should be able to provide in principle an answer as to how the space time was created, and for this reason they should exhibit background independence in some sense. This is an advantage of the loop gravity approach [16] which start from abstract fundamental units (“spin networks”) and then proceeds to construct a dynamical space time out of them.

One may argue that such background independence issues or background changes in string theory can only be tackled in the context of string field theory, which is not developed as yet to a satisfactory level of precision or computational power. One, however, may be less ambitious and attempt to tackle such issues perturbatively at first instance, within a first quantization σ -model approach. The key to this is probably abandoning *conformal invariance* which fixes the background, and use instead *non-critical* (Liouville) strings [22], which is a mathematically consistent way to deal with σ -models away from their conformal (fixed) points on the space of string theories (c.f. figure 3). I will argue below that such a procedure describes consistently background changes in string theory, and implies, under certain circumstances, modified dispersion relations stemming from spontaneous breaking of Lorentz symmetry.

To understand better non-critical strings we should first mention a few things about theory space. This is considered as the space of all possible background target-space-time string configurations, which is therefore an infinite dimensional manifold. The manifold is endowed with a metric, which is nothing other than the Zamolodchikov metric of a conformal field theory [23] given by the appropriate two point function of the vertex operators describing the non-conformal deformations: $\mathcal{G}_{ij} = \langle V_i(z, \bar{z}) V_j(0, 0) \rangle$, where the vertex operators are deforming the σ -model as:

$$S_\sigma = S^* + \int_\Sigma g^i V_i \quad (12)$$

Here S^* is a fixed-point σ -model background action, and g^i are the non-conformal backgrounds/couplings corresponding to the vertex operators V_i . In stringy models of interest to us here $g^i = \{G_{\mu\nu}, \Phi, A_\mu \dots\}$ i.e. is a set of target-space fields.

The non-criticality of the deformation is expressed by the non-triviality of the renormalization-group (RG) β -function of g^i on the world-sheet: $\beta^i = dg^i/d\ln\mu \neq 0$, where μ is a world-sheet scale provided by the world-sheet area [10]. Perturbation theory requires that one lies close to fixed point, which implies that one should work with β^i expandable in power series in the couplings g^i . Quadratic order is sufficient for our

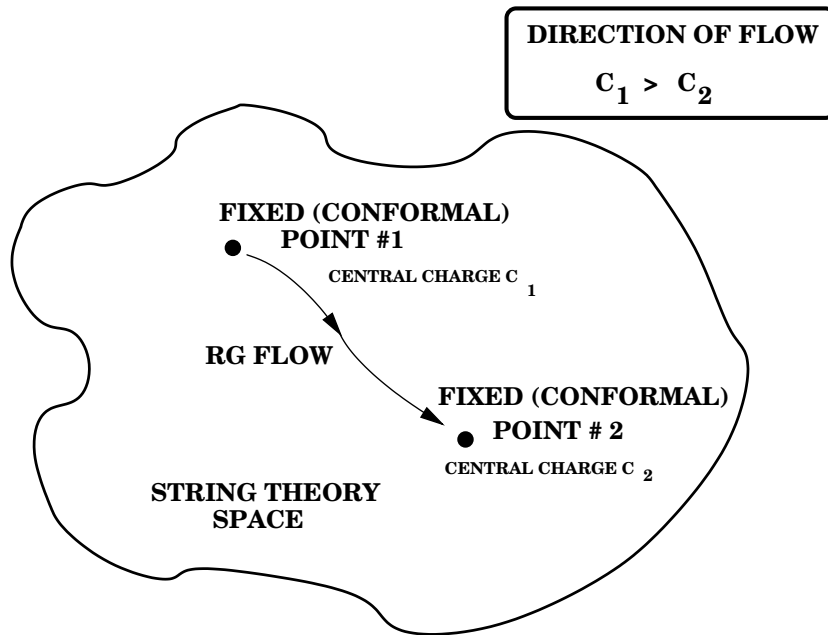


Figure 3. A schematic view of string theory space (which is an infinite dimensional manifold endowed with a (Zamolodchikov) metric). The dots denote conformal string backgrounds. A non-conformal string flows (in a two-dimensional renormalization-group sense) from one fixed point to another (fixed points could even be hypersurfaces in theory space). If the string is a unitary field theory on the world sheet the direction of the flow is towards the fixed point with a lesser value of the central charge.

purposes here, and to this order the β -function read: $\beta^i = y_i g^i + c_{jk}^i g^j g^k + \dots$, where no sum is implied in the first term, y_i is the anomalous dimension, and c_{jk}^i are the operator product expansion (OPE) coefficients.

An important property of the *off-shell* β^i functions of stringy σ -models is their gradient flow form [23, 10] i.e. the fact that they can be derived as g^i -space gradients of a flow function $C[g, t]$. This function is a renormalization-group invariant function and coincides with the running central charge of the theory between fixed points. One has the following relations for its flow along a renormalization-group trajectory, the celebrated Zamolodchikov c -theorem [23]:

$$\frac{\partial C[g, t]}{\partial t} = -\frac{1}{12} \beta^i \mathcal{G}_{ij} \beta^j, \quad \beta^i = \mathcal{G}^{ij} \frac{\partial C[g, t]}{\partial g^j} \quad (13)$$

In unitary σ -model the metric \mathcal{G}_{ij} is positive definite and hence the flow is such that the $C[g, t]$ acts like a thermodynamic H function, decreasing *monotonically* along the direction of the flow. At fixed points $\beta^i = \mathcal{G}^{ij} \frac{\partial C[g, t]}{\partial g^j} = 0$ and taking into account the positive-definiteness of the Zamolodchikov metric one then obtains that the conformal invariance conditions are equivalent to target-space equations of motion for the flow function $C[g, t]$. The latter, thus, plays the rôle of a low-energy effective action for the string theory at hand, which notably includes gravitational interactions. At fixed points the flow function becomes the central charge characterising the two-dimensional conformal field theory. In critical string theories with target space-time interpretation the flow function becomes 26 for bosonic strings (or 10 in superstrings).

For non-unitary σ -models, such as stringy σ -models with time-like X^0 fields, and time-like dilaton couplings, the flow may not be monotonic all the way, but one may

still expect [24] an overall decrease of the running central charge from one fixed point to the other. This is due to the information “loss” beyond the ultraviolet cutoff of the underlying world-sheet quantum field theory. In fact the central charge counts degrees of freedom of the system, and hence there must be an inherent irreversibility if there is a cutoff scale. In non-critical strings with time-like fields, the running central charge does make oscillations before settling to a fixed point [25, 10]. This will become important later on (section 4) when we discuss cosmology in this context.

Let me now argue how non-critical strings become consistent world-sheet (two-dimensional) field theories upon Liouville dressing [22]. The Liouville mode φ seizes to decouple in a non-critical string which is characterized by a (RG running) central charge deficit $Q^2 \equiv \frac{1}{3}(C[g, t] - c^*)$ caused by departure from criticality due to a given deformation (for strings with a target-space interpretation $c^* = 25$ (9 for superstrings)).

A detailed analysis [22, 10, 25] shows that the Liouville dressed theory is described by a σ -model action:

$$\begin{aligned} S_\varphi &= Q^2 \int_\Sigma \partial\varphi \bar{\partial}\varphi + \int_\Sigma Q^2 R^{(2)} + \int_\Sigma \lambda^i(\varphi) V_i , \\ \lambda^i(\varphi) &= g^i e^{\alpha_i Q\varphi} + \frac{\pi}{Q + 2\alpha_i} c_{jk}^i g^j g^k Q\varphi e^{\alpha_i Q\varphi} , \\ \alpha_i (\alpha_i + Q) &= -(\Delta_i - 2) , \text{ no sum over } i \end{aligned} \quad (14)$$

where $\lambda^i(\varphi)$ are the Liouville dressed couplings and the gravitational anomalous dimension α_i is chosen in such a way so that the conformal dimension of the dressed deformation operator $\lambda(\varphi)V_i$ becomes one (marginal). Note that this marginality is exact, including interactions (c_{jk}^i etc in the β -function) and this constitutes the main rôle of the Liouville dressing, *the restoration of conformal symmetry*, in such a way that the non-critical, RG flowing σ -model, becomes conformal at the expense of the introduction of the Liouville mode φ in the path integral.

From the quadratic equation that the gravitational anomalous dimensions satisfy (14) it becomes evident that there are in general two solutions:

$$\alpha_i^\pm = -\frac{Q}{2} \pm \sqrt{\frac{Q^2}{4} + \text{sgn}(Q^2 - 25)(\Delta_i - 2)} \quad (15)$$

Usually in Liouville theory (central charge $c < 1$) one keeps only the $+$ solution, given that α_- corresponds to states that “do not exist” in the sense of not having a well-defined (bounded, normalizable) behaviour in the semiclassical limit where the central charge $|c| \rightarrow +\infty$. However in supercritical string theory (central charge $c > 25$) this is not the case and one may, and in some circumstances does, keep both dressings. We shall encounter a situation like this in section 4, when we discuss specific examples of Liouville strings.

Upon redefining the Liouville mode

$$\phi \equiv Q\varphi \quad (16)$$

one may arrive at *Liouville-generalized conformal invariance conditions* [22]:

$$\ddot{\lambda}^i + Q(t)\dot{\lambda}^i = -\text{sgn}(C[g, t] - 25)\beta^i(\lambda^i(\phi)) \quad (17)$$

where the overdot denotes differentiation with respect to the normalized Liouville mode ϕ (16).

From (14) it is obvious that the Liouville mode plays the rôle of an extra target-space dimension, which is *time-like* if the string is supercritical [26] $Q^2 > 0$ and *spacelike* if the string is subcritical. It is a general property of Liouville strings that during their

flows they remain either subcritical or supercritical [10, 25]. In what follows we shall concentrate on deformations that induce supercriticality of the string, in which case the Liouville mode may be identified with the target time [10]. In fact, what one identifies with the target time is the world-sheet zero mode of the normalized field

$$\phi \equiv Q\varphi \rightarrow t = \text{target time} \quad (18)$$

In what follows I shall describe the most important physical consequences of such an identification.

3.3. Absence of S -matrix and Non-Equilibrium Dynamics in non-critical Strings with Time as the Liouville mode

From (17) it becomes clear that the restoration of conformal symmetry leads to dynamical equations for the background fields of the non-critical σ -model that have a “frictional” form, the damping being provided by the (square root of the) central charge deficit Q^2 . Although the sign of Q^2 is fixed, however the sign of its square root Q is not determined in Liouville strings, and as we shall see in section 4, one may encounter situations [27, 28] in which $Q(t)$ evolves in such a way so as to change sign (in such a case the string passes through a metastable critical point in theory space).

From (14) it also becomes clear that after the identification (18) there will be a dilaton coupling in target space (defined as the background coupling of the world-sheet curvature term in the σ model action) $\Phi \sim Q(t)t$ which will contain linear dependences on target time for Q constant, but in general will be a complicated function of time. String theories in such time-like supercritical backgrounds can be formulated as consistent conformal field theories only for some special cases of constant Q (linear dilatons) [26] but the corresponding S -matrix aspects are problematic. One can define factorisable and modular invariant amplitudes (S -matrix) in such a case only for certain *discrete* constant values of Q , as required for consistency of the mapping of the corresponding conformal theory to a Coulomb gas representation with appropriate screening charges at infinity [26].

Nevertheless it is possible to consider more general situations, with continuously varying Q , provided that one abandons the concept of a factorisable scattering amplitude [10]. To understand this latter point let us consider a generic correlation function among n vertex operators V_i of a Liouville string:

$$\mathcal{A}_n = \langle V_{i_1} \dots V_{i_n} \rangle_g = \int \mathcal{D}\phi \mathcal{D}X e^{-S^* - \int_{\Sigma} d^2\sigma \lambda^i(\phi) V_i + \partial\phi \bar{\partial}\phi - Q \int_{\Sigma} d^2\sigma \phi R^{(2)}} V_{i_1} \dots V_{i_n} \quad (19)$$

where ϕ is the normalized Liouville mode (16), and Q^2 denotes the central charge deficit, quantifying the departure of the non-conformal theory from criticality [22].

A detailed analysis [10] shows that, upon performing the world-sheet zero-mode ϕ_0 integration of the Liouville mode ϕ in (19), one obtains that the dominant contributions to the path integral are proportional to

$$\mathcal{A}_n \propto \Gamma(-s) \langle \dots \rangle' \quad (20)$$

where the prime in $\langle \dots \rangle'$ indicates the absence of world-sheet zero mode $\phi_0 \sim \ln A$ of the Liouville field ϕ , with A the world-sheet area (in units of the ultraviolet cutoff of the σ -model), $s = \sum_i \frac{\alpha_i}{\alpha} + \frac{Q}{\alpha}$ and α_i are the Liouville anomalous dimensions (14), and α is the anomalous dimension of the so-called identity operator $\alpha = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + 8}$.

In order to compute the integral for arbitrary s (and hence Q) it is necessary to make an analytic continuation to positive integer $s = n^+ \in \mathbb{Z}^+$ which immediately calls

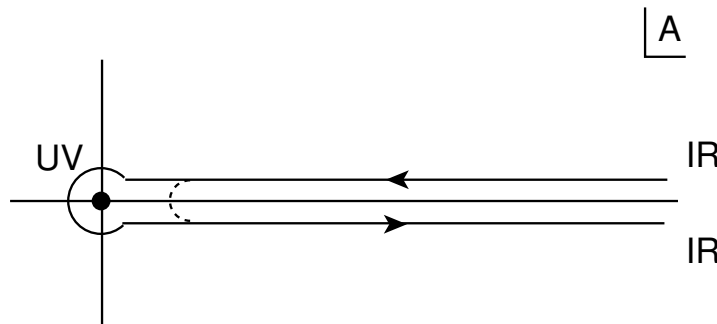


Figure 4. Contour of integration for a proper definition of Liouville field path integration. The quantity A denotes the (complex) world-sheet area, which is identified with the logarithm of the Liouville (world-sheet) zero mode. This is known in the literature as the Saalschutz contour, and has been used in conventional quantum field theory to relate dimensional regularization to the Bogoliubov-Parasiuk-Hepp-Zimmermann renormalization method. Upon the interpretation of the Liouville field as target time, this curve resembles closed-time-paths in non-equilibrium field theories.

for regularization of the prefactors $\Gamma(-s)$. This can be achieved [10, 32] by representing the integral over the Liouville zero mode by a steepest-descent contour of ϕ_0 as indicated in fig. 4. The interpretation of the Liouville zero mode as the target time [10] implies a direct analogy of this contour with *closed time like paths* in *non-equilibrium* field theories [29].

It can be seen [10] that the world-sheet ultraviolet limit of the contour $A \rightarrow 0^+$ is ill defined and a regularization is needed (dashed line in the contour of fig. 4). This can be inferred by considering infinitesimal Weyl shifts of the world-sheet metric (which is integrated over) in the correlator (19). Such a divergent behaviour implies that the generic Liouville correlator (19) is not an S-matrix amplitude, as conventionally assumed in critical strings, but rather a \$ matrix element connecting asymptotic density matrices [30] $\rho_{\text{out}} = \$ \rho_{\text{in}}$ with $\$ \neq SS^\dagger$ as a result of the world-sheet ultraviolet divergences. From a target space time view point small world-sheet areas may be seen as an infrared limit of a target space theory, where the strings shrink to their point-like limits. Thus, in this picture, the infrared divergences in target space are held responsible for the failure of factorisability of the Liouville string \$ matrix.

It is this feature that, together with the *irreversibility* of the world-sheet renormalization group flow function, expressed by the c-theorem (13), makes manifest the *non equilibrium nature* of non-critical string theory. There is thus a direct analogy of Liouville strings with open systems in which there is an entropy change (here the entropy is associated with the flow function $C[g, t]$ [10]). Further support of the non-equilibrium nature comes from the modification of the evolution equation of the density matrix of string matter moving in non-critical string backgrounds. Renormalization-group considerations on the σ -model theory, and in particular renormalization-group invariance of physical target space quantities, imply the following RG flow equation for the density matrix of low-energy string matter ρ [10, 32]:

$$\frac{\partial \rho}{\partial t} = i[\rho, H] + : \beta^i \mathcal{G}_{ij}[g^j, \rho] : \quad (21)$$

where the quantization is achieved in this framework by summing up world-sheet topologies. In fact it can be shown that canonical quantization in the case of Liouville strings is possible, given that the generalized conformal invariance conditions (17) satisfy the Helmholtz conditions for being derived from an off-shell Lagrangian in g^i space [10].

Note the similarity of (21) with the evolution of open dynamical systems. Thus we see that the non-criticality of the string, expressed by the non-vanishing of the world-sheet RG β -functions, leads to the existence of “environment” for the subsystem of the low-energy string matter. In fact an important comment is in order here: there are quantum ordering ambiguities of the last term due to the fact that $\beta^i \mathcal{G}_{ij}$ are complicated power series in the operators g^i which do not commute with ρ . One might have thought that these ambiguities may be resolved by postulating an ordering that respects formally the Lindblad form of the interaction of the string matter with its non-critical string environment. However, things may not be that simple. It must be noted that there is a hidden non-linearity in the evolution equation (21) due to the string wavefunction dependence of the Zamolodchikov metric $\mathcal{G}_{ij} \sim \langle V_i V_j \rangle$. When one writes the perturbative expansion $\beta^i \mathcal{G}_{ij} = \sum_{i_n} c_{ji_1 \dots i_n} g^{i_1} \dots g^{i_n}$ this dependence is hidden in the coefficients $c_{i_1 \dots i_n} \sim \langle V_{i_1} \dots V_{i_n} \rangle_*$ where $\langle \dots \rangle_*$ denote free Liouville string correlators. Such non-linearities may imply significant deviations from standard linear Lindblad environmental entanglement [33]. The situation should be examined case by case unfortunately, as it seems that general statements on this issue cannot be made at this stage.

3.4. Modified Dispersion Relation in Liouville Strings and Lorentz Invariance Violation

To study the induced modifications on the dispersion relations of string excitations in the case of non critical deformations we concentrate on the generic Liouville dressing (14). Upon the identification of the Liouville mode with time (18) we may rewrite this relation as:

$$\lambda^i(\phi) = \lambda^i(t) e^{i(\alpha_i + \Delta\alpha_i)t} \quad (22)$$

where $\Delta\alpha_i$ depends on the OPE coefficients c_{jk}^i which encode the non-critical interactions of the string probe with the “environment”. In string theory the OPE, which are related to string target-space scattering amplitudes, are power series in $\alpha' |\vec{k}|^2$ for closed strings, and $\sqrt{\alpha'} |\vec{k}|$ for open strings. Let us concentrate in the case of closed strings for definiteness, and also for physical concreteness, given that closed strings include gravity. We restrict ourselves to *massless modes* for our purposes below. To order g^2 (i.e. close to a fixed point), where we restrict ourselves for our purposes in this article, the OPE are assumed of order $\alpha' |\vec{k}|^2$. On the other hand, for vertex operators proportional to $e^{ik_\mu X^\mu}$ (in Fourier space), describing the string excitation spectrum in flat space times, the anomalous dimensions $\alpha_i \sim |\vec{k}|^2$. This can be found from (15) by noting that the central charge deficit Q^2 is determined from the c-flow theorem (13) $Q^2(t) \sim \int^t \beta^i \mathcal{G}_{ij} \beta^j = \mathcal{O}(|\vec{k}|^4)$, since $\beta^i = \mathcal{O}(|\vec{k}|)$ for reasons stated previously.

Thus, for massless closed string excitations one has the estimate:

$$\alpha_k + \Delta\alpha_k \sim \ell_s |\vec{k}| \left(1 + \mathcal{O}(|\vec{k}|^2 \ell_s^2)\right)^{1/2} = \ell_s |\vec{k}| + \mathcal{O}(\ell_s^3 |\vec{k}|^3) \quad (23)$$

The complete Liouville-dressed vertex operator, therefore, describing the massless string excitation reads:

$$\lambda^k(t) V_k \sim g^k e^{i(\alpha_k + \Delta\alpha_k)t} e^{i\vec{k} \cdot \vec{X}} \sim e^{i(|\vec{k}| + \mathcal{O}(\ell_s^2 |\vec{k}|^2))t + i\vec{k} \cdot \vec{X}} \quad (24)$$

The coefficient of the Liouville mode (time) is defined as the energy of the probe, and hence from (24) one obtains a modified dispersion relation for massless string excitations [10]:

$$E \sim |\vec{k}| + \eta \ell_s |\vec{k}|^2 \quad (25)$$

where the coefficient η and its sign is to be determined in specific models of non-critical strings, and the string scale $\ell_s = \sqrt{\alpha'}$ may be taken to be the Planck scale M_P^{-1} of quantum gravity (but, we repeat again, in modern versions of string theory where our world is viewed as a string membrane [21] ℓ_s is an arbitrary free parameter, which is in general different from the four-dimensional Planck scale M_P^{-1}).

From the modified dispersion one obtains a refractive index for the massless probe in the sense of its group velocity not being equal to one:

$$\frac{\partial E}{\partial |\vec{k}|} = 1 + 2\eta\ell_s|\vec{k}| + \dots \quad (26)$$

which implies violation of Lorentz invariance. The latter should not come as a surprise given the non-equilibrium stochastic nature of the non-critical string (c.f. (21)) which resembles an open system in which string matter propagates in a ‘stochastic’ dissipative way (c.f. (17)).

However, in our approach [10] the Lorentz violation is *spontaneous* in the sense that the full string theory may be critical, and the non-criticality is only a result of restricting oneself in an effective low-energy theory (“ground state” not respecting the symmetry). In this last sense the presence of the Liouville mode is a *collective* description of “environmental” effects associated with degrees of freedom essentially unobserved by a local observer [10].

The precise nature of such degrees of freedom can be unveiled in realistic concrete examples of non-critical strings with physical significance. We examine two such examples in the next section.

4. Concrete Physical Examples of Liouville Strings

4.1. A Liouville string model for Space Time foam

We describe below a toy model for stringy space-time foam [10], consisting of a bulk space time “punctured” with D(irichlet)-0-brane point-like defects. The latter are solitonic states in string theory [21], and as such they are not simply structureless point-like objects, but have substructure in the sense of an infinity of internal degrees of freedom corresponding to the massive string states. Once a closed string, representing light string matter in the model, is scattered off the defects, the latter recoil and distort the surrounding space time. The distortion is expressed through the dynamical formation of (unstable) horizons, the interior of which is found to have a different refractive index from the exterior flat Minkowski space time. The situation is schematically depicted in figure 5.

The mathematical formalism is based on a conformal field theory treatment of recoil in D-brane theory [34] which allows for changes of σ -model backgrounds. This can be achieved by the so-called logarithmic conformal field theories (LCFT), which lie on the border line between conventional conformal field theories and general two-dimensional field theories, and can still be classified by conformal data. Below we review briefly the main results.

A D-particle, recoiling with velocity v^i , and initially positioned at y^i , can be viewed as a D-particle with open string excitations attached to it (c.f. middle figure 5), which from a σ -model point of view are described by adding to the free string σ -model the following boundary (world-sheet disk) deformations

$$\mathcal{V}_{\text{rec,rest}} = \int_{\partial\Sigma} \left(\epsilon^2 y_i \theta_\epsilon(X^0) \partial_n X^i + \epsilon v_i X^0 \theta_\epsilon(X^0) \partial_n X^i \right) \quad (27)$$

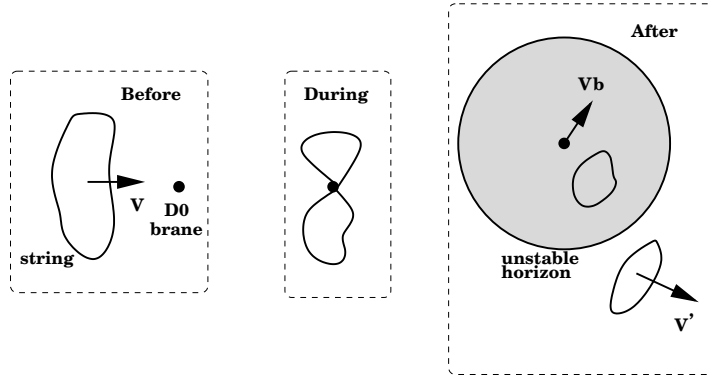


Figure 5. A schematic representation of D-particle foam; closed strings scatter off D0 brane defects in space time. The scattering distorts the surrounding space time, causing the formation of unstable horizons with non-trivial refractive index in their interior. These structures capture part of the propagating string matter, thereby leading to stochastic medium behaviour, and hence a space-time foam situation.

where ∂_n is a normal world-sheet boundary derivative, and we assumed that the D-particle was initially *at rest*. These operators form a LCFT pair of operators [34]. In the above formula $\theta_\epsilon(X^0) \sim \theta(X^0)e^{-\epsilon X^0}$ for $X^0 > 0$ is a regularized Heaviside operator, with $\theta(X^0)$ the ordinary Heaviside function. The coordinate X^0 obey Neumann boundary conditions on the world-sheet boundary $\partial\Sigma$, whilst X^i , i spatial index, obey Dirichlet boundary conditions, and express the fact that the D-particle defect can trap strings with their ends attached on it (c.f. figure 5). In the limit $\epsilon \rightarrow 0^+$ which we consider here the first term in (27) is subleading and from now on we ignore it.

We next extend the discussion to motion of string matter through a gas of moving D-particles [35], as is likely to be the case for a laboratory on Earth, e.g., if the D-particle foam is comoving with the Cosmic Microwave Background (CMB) frame. Assuming this to be moving with three-velocity \vec{w} relative to the observer, the recoil deformation takes the following form to leading order in $\epsilon \rightarrow 0^+$:

$$\mathcal{V}'_{\text{rec}} = \int_{\partial\Sigma} \theta_\epsilon(-X_w^0) \gamma_w w^i \partial_n X_w^i + \int_{\partial\Sigma} \theta_\epsilon(X_w^0) \gamma_w (w^i + \epsilon v^i(w)) \partial_n X_w^i \quad (28)$$

where the suffix w denotes quantities in the boosted frame. The recoil velocity $v^i(w)$ depends in general on w , and is determined by momentum conservation during the scattering process, as discussed in [34]. The main novelty in the $w \neq 0$ case is that now there are two σ -model operators in (28).

The deformations (27) and (28) are relevant world-sheet deformations in a two-dimensional renormalization-group sense, with anomalous scaling dimensions $-\epsilon^2/2$ [34]. Their presence drives the stringy σ -model non-critical, and requires dressing with the Liouville mode ϕ . As mentioned previously, in section 3, in Liouville strings there are two screening operators $e^{\alpha_\pm \phi}$, where the α_\pm are the Liouville-string anomalous dimensions given by:

$$\alpha_\pm = -\frac{Q}{2} \pm \sqrt{\frac{Q^2}{2} + \frac{\epsilon^2}{2}} \quad (29)$$

and the central charge deficit Q was computed in [10] with the help of the c-theorem (13), and found to be of higher order than ϵ^2 . Hence $\alpha_\pm \sim \pm\epsilon$.

The α_- screening operator is sometimes neglected because it corresponds to states that do not exist in Liouville theory. However, this is not the case in string theory,

where one should keep both screenings as above. This is essential for recovering the correct vanishing-recoil limit in the case of infinite D-particle mass. The Liouville-dressed boosted deformation then reads:

$$\begin{aligned} \mathcal{V}'^L_{\text{rec}} &= \int_{\partial\Sigma} e^{\alpha-\phi} \theta_\epsilon(-X_w^0) \gamma_w w^i \partial_n X_w^i + \\ &\int_{\partial\Sigma} e^{\alpha+\phi} \theta_\epsilon(X_w^0) \gamma_w (w^i + \epsilon v^i(w)) \partial_n X_w^i \end{aligned} \quad (30)$$

Using Stokes' theorem, and ignoring terms that vanish using the world-sheet equations of motion, one arrives easily at the following bulk world-sheet operator:

$$\mathcal{V}'^L_{\text{rec}} = \int_{\Sigma} e^{\epsilon\phi} \theta_\epsilon(X_w^0) \epsilon^2 \gamma_w v^i(w) \partial_a X_w^i \partial^a \phi + \dots, \quad a = 1, 2, \quad (31)$$

where the \dots denote terms subleading as $\epsilon \rightarrow 0^+$ as we shall explain below. Notice the cancellation of the terms proportional to w due to the opposite screening dressings. Recalling that the regularised Heaviside operator $\theta_\epsilon(X) = \theta_0(X) e^{-\epsilon X}$, where $\theta_0(X)$ is the standard Heaviside function, we observe that one can identify the *boosted time coordinate* X_w^0 with the Liouville mode ϕ : $\phi = X_w^0$ [10]. At long times after the scattering, the Liouville-dressed theory leads to target-space metric deformations of the following form to order ϵ^2 :

$$G_{\phi i} = \epsilon^2 \gamma_w v^i(w) \phi \quad (32)$$

As explained in detail in [34, 10], at the long times after the scattering event when the σ -model formalism is valid, one has $\epsilon^2 \phi_0 \sim 1$: ϵ and the world-sheet zero mode of ϕ , ϕ_0 , are not independent variables, as ϵ is linked with the world-sheet renormalization-group scale.

The Lorentz breaking effects are proportional to $v^i(w)$, which is determined by momentum conservation and hence is independent of w in the non-relativistic limit. In the case of photon scattering off D-particles, one would have $v^i(w) = \sqrt{(1+|w|)/(1-|w|)} v^i(0)$. *Our form of Lorentz violation exhibits Galilean invariance* for small w . This is an important feature which differentiates our Liouville foam model from generic Lorentz-violating models of space time foam considered in the literature [36]. For our purposes, the main effect of our D-particle model for space-time foam is the modification of the dispersion relation for an energetic particle, which causes it to propagate more slowly than a less energetic particle.

The formation of horizons, mentioned earlier, and depicted in figure 5, is obtained upon considering quantum fluctuation effects, which express the probability of finding a particular defect configuration as a fluctuation around a mean value. Formally, one can evaluate quantum fluctuations in the recoil process by making an appropriate resummation over the world-sheet genera, which lead to stochastic probability distribution functions in the σ -model path integral [34]. Performing appropriate coordinate transformations, which from a σ -model view point amount to redefining path integral variables, and averaging over quantum fluctuations one can map the metric (32) in the limit $w = 0$ into one which has an expanding horizon surrounding the defect at: $r_{\text{horizon}}^2 = t^2/b'(E)^2$, with $b'(E)^2 = 4g_s^2 \left(1 - \frac{255}{18} g_s^2 \frac{E_{\text{kin}}}{M_D}\right) + \mathcal{O}(g_s^6)$, $E_{\text{kin}} (\equiv E)$ is the kinetic recoil energy of the non-relativistic D-particle, $g_s < 1$ is the string coupling, assumed weak for the validity of the σ -model perturbative expansion, and $M_D = M_s/g_s$ is the D-brane mass scale with M_s the string scale. Often one assumes M_s close to the four dimensional Planck mass of $M_P \sim 10^{19}$ GeV, although in general M_s, g_s are

parameters which can be constrained by data [14]. Inside the horizon, the space-time metric has the form:

$$ds_{\text{inside}}^2 = \frac{b'(E)^2 r^2}{t^2} dt^2 - \sum_{i=1}^3 dx_i^2 : \quad r^2 = \sum_{i=1}^3 x_i^2, \quad (33)$$

and one can match this space-time with a flat external Minkowski space-time, as explained in detail in [10]. The internal ‘bubble’ metric configuration (33) violates the positive energy conditions and hence is *unstable*. Thus a ‘breathing bubble’ picture emerges from such a world-sheet path integration of the Liouville zero mode [10], in which the average lifetime of the bubble is Planckian, during which the bubble expands to its maximum size, which is of order the Planck length, and then recontracts.

We have identified three possible experimental implications of the formation of such quantum space-time bubbles. *First*, low-energy particles may get trapped inside the bubble, and their absorption within the space-time foam provides an explicit mechanism for the apparent loss of information accessible to external observers. *Secondly*, in the case of electrically-charged particles, their non-uniform (spiral) motion inside the bubble causes the emission of radiation, which may escape in the form of photons. Since the bubble interior has a non-trivial refractive index [10], these particles exhibit transition radiation, i.e., the emission of photons accompanying an electrically-charged particle when it crosses an interface separating two media with different refractive indices. A fraction of this radiation escapes from the bubble, yielding photons that accompany the charged particle. *Thirdly*, propagation inside the bubble causes the particle to travel more slowly, as seen by an external observer. This effect is equivalent to the retardation induced by the modification (32) of the space-time metric, as the D-brane defect recoils when struck by the propagating particle.

The refractive index in this D-brane foam can be computed as [10]

$$v = 1 - \mathcal{O}\left(\frac{g_s E}{M_s}\right) \quad (34)$$

where E is the kinetic energy transfer of the matter probe during the collision with the D-particle defect. The index (34) is always *subluminal*, thereby differentiating the model from certain models of loop gravity where superluminal propagation also occurs [15]. The subluminal nature is due to specific stringy properties of the model, in particular it stems from the fact that the effective action describing the recoil excitations is of Born-Infeld form of non-linear “electrodynamics” [34], and hence there is a limiting (light) velocity for signal propagation.

For astrophysical and other tests of these string-inspired dispersion relation see [6]. We should mention, though, that the non-critical string model of space time foam of [10], despite its minimal Planck-scale suppression of the effects, seems to avoid severe constraints, especially from atomic physics experiments [36, 35], which seem to exclude the loop-gravity minimal suppression models. Thus, for this model of foam, GRB arrival-time tests [14] seem, as yet, to provide the most sensitive probes.

4.2. A Liouville String approach to Colliding Branes Cosmology: Inflation, and Supersymmetry-Breaking/Vacuum-Energy Hierarchy

As mentioned in the introduction, there has been recent exciting evidence from astrophysical observations on high redshift type I Supernovae [2] that our Universe is currently accelerating. The evidence has been seconded by cosmic microwave background data [3] pointing towards total flatness of our Universe $\Omega_{\text{total}} = 1$, a result

which, when combined with the estimate for the total matter contribution (including dark matter) $\Omega_M \sim 0.3$, implies that there is a dark energy component in our Universe which is 70 % of the total energy density. This is in agreement with the Supernovae data implying acceleration of the Universe, and implies that the Universe is currently in a phase where this dark energy component begins to take over. These data are in agreement with naive cosmological constant in the Universe (de Sitter), but this issue present an enormous theoretical challenge, due to the cosmic horizon problem (1) and its incompatibility with the definition of a scattering matrix [8], and hence string theory.

As we have seen in section 3, Liouville non-equilibrium strings, are not characterised by a proper S-matrix, nevertheless they could be quantized consistently on the world-sheet. It is therefore legitimate to argue [38] that, if our Universe has a cosmological constant, then the only way to quantize it within string theory is to postulate some sort of Liouville string cosmology. One, however, may actually attempt to go one step further and try to find cosmological non-critical string models which eventually asymptote (in time) to equilibrium (critical string) cosmologies. In such models, which resemble *quintessence models* [37] in conventional cosmology, there is an initial cause for departure from criticality (conformality of the respective σ -model, i.e. the presence of a central-charge deficit $Q^2 > 0$), which imply that the Universe undergoes a non-equilibrium, and then a relaxation phase, which lasts until today. Eventually the central charge deficit vanishes, and the string theory reaches its critical equilibrium situation. This allows an appropriate definition of asymptotic states and an S -matrix, since in such models one eventually exits from the de Sitter accelerating phase.

In what follows I will give a concrete example of such a non-equilibrium cosmology. The model involves two colliding branes worlds, one of which is assumed to be our world. We shall be very brief in our exposition. For details we refer the reader to [28] where the model is described in some detail. I will argue below that the model produces inflation and a relaxing to zero cosmological “constant” hierarchically small as compared to the supersymmetry breaking (TeV) scale. Supersymmetry breaking is induced by compactification of the brane worlds on magnetized tori. The crucial ingredient is the non-criticality (non conformality) of string theory on the observable brane world induced at the collision, which is thus viewed as a cause for departure from equilibrium in this system. The hierarchical smallness of the present-era vacuum energy, as compared to the SUSY breaking scale, is attributed to relaxation phenomena.

The model consists of two five-branes of type IIB string theory, embedded in a ten dimensional bulk space time. Two of the longitudinal brane dimensions are assumed compactified on a small torus, of radius R . In one of the branes, from now on called *hidden*, the torus is *magnetized* with a constant magnetic field of intensity H . This amounts to an effective four-dimensional vacuum energy contribution in that brane of order: $R^2 H^2 > 0$. Notice that such compactifications provide alternative ways of breaking supersymmetry [39], which we shall make use of in the current article. In scenaria with two branes embedded in higher-dimensional bulk space times it is natural to assume (from the point of view of solutions to bulk field equations) that the two branes have *opposite* tensions. We assume that before the collision the visible brane (our world) has positive tension $V_{\text{vis}} = -V_{\text{hidd}} > 0$. The presence of opposite tension branes implies that the system is not stable, but this is O.K. from a cosmological view point.

For our purposes we assume that the two branes are originally on *collision course* in the bulk, with a relative velocity $u \ll 1$ for the validity of the σ -model perturbation theory, and to allow for the model to have predictive power. The collision takes place at a given time moment. This constitutes an event, which in our scenario is identified

with the *initial cosmological singularity* (big bang) on the observable world. We note that similar scenaria exist in the so-called ekpyrotic model for the Universe [40]. It must be stressed, though, that the similarity pertains only to the brane-collision event. In our approach the physics is entirely different from the ekpyrotic scenario. The collision is viewed as an event resulting in non-criticality (departure from conformal invariance) of the underlying string theory, and hence in non-vanishing β functions at a σ -model level. On the contrary, in the scenario of [40] the underlying four-dimensional effective theory (obtained after integration of the bulk extra dimensions [40, 41]) is assumed always critical, satisfying classical equations of motion, and hence vanishing σ -model β functions. This latter property leads only to contracting and *not expanding* four-dimensional Universes according to the work of [41], which constitutes one of the main criticisms of the ekpyrotic universe. On the other hand, in our non-critical description of the collision we do not assume classical solutions of the equations of motion, neither specific potentials associated with bulk branes, as in [40].

The physics of our colliding worlds model can be summarized as follows: During the collision one assumes electric current transfer from the hidden to the visible brane, which results in the appearance of a magnetic field on the visible brane. We also assume that the entire effect is happening very slowly and amounts to a slow flow of energy and current density from the hidden to the visible brane (our world). In turn, this results in a positive energy component of order $H^2 R^2$ in the vacuum energy of the visible brane world.

At the moment of the collision the conformal invariance of the σ -model describing (stringy) excitations on the observable brane world is spoiled, thereby implying the need for Liouville dressing [22, 10]. This procedure restores conformal invariance at the cost of introducing an extra target space coordinate (the Liouville mode ϕ), which in our model has time-like signature. Hence, initially, one faces a two-times situation. We argued [28], though, that our observable (cosmological) time X^0 parametrizes a certain curve, $\phi = \text{const.}$, $X_0 + \text{const.}$, on the two-times plane (X_0, ϕ) , and hence one is left with one physical time.

The appearance of the magnetic field on the visible brane, on the dimensions $X^{4,5}$, is described (for times long after the collision) within a σ -model superstring formalism by the boundary deformation: $\mathcal{V}_H = \int_{\partial\Sigma} A_5 \partial_\tau X^5 + \text{supersymm. partners}$, where $A_5 = e^{\varepsilon X^0} H X^4$, and ∂_τ denotes tangential σ -model derivative on the world-sheet boundary. This σ -model deformation describes open-string excitations attached to the brane world (c.f. fig. 6). The presence of the quantity $\varepsilon \rightarrow 0^+$ reflects the *adiabatic* switching on of the magnetic field after the collision. It should be remarked that in our approach the quantity ε is viewed as a world-sheet renormalization-group scale parameter [28]. In addition to the magnetic field deformation, the σ -model contains also boundary deformations describing the ‘recoil’ of the visible world due to the collision: $\mathcal{V}_{\text{rec}} = \int_{\partial\Sigma} Y_6(X_0) \partial_n X^6 + \text{supersymm. partners}$, where $Y_6(X_0) = u X^0 e^{\varepsilon X^0}$, ∂_n denotes normal σ -model derivative on the world-sheet boundary, and u is the recoil velocity of the visible brane world.

The presence of the exponential factors $e^{\varepsilon X^0}$ in *both* the magnetic field and recoil deformations implies a small but negative world-sheet anomalous dimension $-\frac{\varepsilon^2}{2} < 0$, and hence the relevance of both operators from a renormalization-group point of view. The induced central-charge deficit Q^2 , which quantifies the departure from the conformal point of the pertinent σ -model, can be computed by virtue of the Zamolodchikov’s c-theorem [28]: $\frac{d}{d\mathcal{T}} Q^2 \sim -\frac{H^2 + u^2}{\mathcal{T}^2} \rightarrow Q^2(\mathcal{T}) = Q_0^2 + \frac{H^2 + u^2}{\mathcal{T}}$, $\mathcal{T} = \ln|L/a|^2$, with $L(a)$ the world-sheet infrared (ultraviolet) cutoff scale. As discussed in detail in [28], the correct scaling behaviour of the operators necessitates the identification $\mathcal{T} \sim \varepsilon^{-2}$

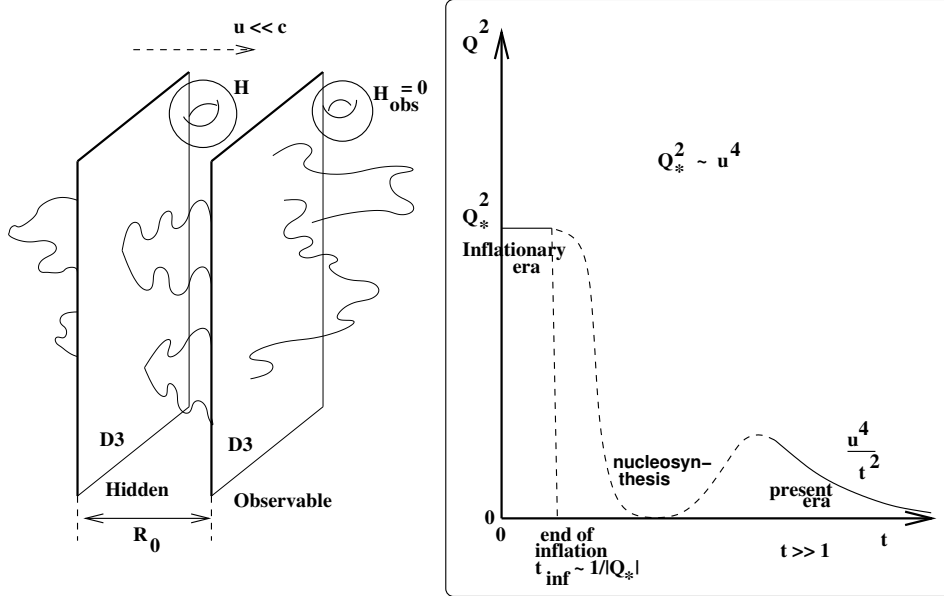


Figure 6. A scenario involving two colliding type-II 5-branes which provides inflation and a relaxation model for cosmological vacuum energy.

which we assume from now on. The quantity $Q_0^2 = Q^2(\infty)$ is the equilibrium vacuum energy density, which we take to be zero $Q_0^2 = 0$ due to an assumed cancellation between Anti-de-Sitter bulk and brane vacuum energies after the collision. We shall come back to this point later on.

The non-conformal deformed σ -model can become conformal by Liouville dressing [22, 28], as discussed in section 3: $A_5(X_0, X_4, \phi) = HX^4 e^{\varepsilon X^0 + \alpha\phi}$, $Y_6(X_0, \phi) = uX^0 e^{\varepsilon X^0 + \alpha\phi}$, where $\alpha \sim \varepsilon$ are the Liouville anomalous dimensions [22] and ϕ is the normalized Liouville mode, whose zero-mode is related to the renormalization-group scale \mathcal{T} as: $\phi = Q(\mathcal{T})\mathcal{T}$. From the work of [39] it becomes clear that the coupling constant H is associated with supersymmetry-breaking mass splittings. This has to do with the different way fermions and bosons couple to an external magnetic field. The mass splittings squared of an open string are in our case [28]:

$$\Delta m_{\text{string}}^2 \sim H e^{\alpha\phi + \varepsilon X^0} \Sigma_{45} \quad (35)$$

The so-obtained mass splittings are constant upon the requirement that the flow of time X^0 and of Liouville mode ϕ are correlated in such a way that

$$\varepsilon X^0 + \varepsilon\phi/\sqrt{2} = \text{constant} , \quad (36)$$

or at most slowly varying. Notice that deviations from the condition (36) would result in very large negative-mass squares, which are clearly unstable configurations. Hence, the identification (36) seems to provide a resolution of this problem. To ensure the phenomenologically reasonable order of magnitude of a TeV scale, one must assume very small [39] $H \sim 10^{-30} \ll 1$ in Planck units. Note also that parametrizing the condition (36) as $X^0 = t$, $\phi_0 = \sqrt{2}t$, and taking into account that, for convergence of σ -model path integration, it is formally necessary to work with Euclidean signature X^0 [34], the induced metric on the hypersurface (36) in the extended space time acquires a Minkowskian-signature Robertson-Walker form: $ds_{\text{hypersurf}}^2 = -(d\phi_0)^2 + (dX^0)^2 + \dots = -(dt)^2 + \dots$ where \dots denote spatial parts. In [28], where we refer the interested reader,

we have given some arguments on a *dynamical stability* of the condition (36) in the context of Liouville strings. Physically, one may interpret this result as implying that a time-varying magnetic field induced by the collision implies back reaction of strings onto the space time in such a way that the mass splittings of the string excitation spectrum, as a result of the field, are actually stabilized.

We now notice that in our case the dilaton field is $\Phi = Q\phi = Q^2\varphi \sim (H^2 + u^2)$, that is, one faces a situation with an asymptotically constant dilaton. This is a welcome fact, because otherwise, the space-time would not be asymptotically flat, and one could face trouble in appropriately defining masses. In the case of a constant dilaton the vacuum energy is determined by the central-charge deficit Q^2 , which in our case is:

$$\Lambda = \frac{R^{2n}}{\phi_0^2} (H^2 + u^2)^2 \quad (37)$$

where $\phi_0 = t$ is the world-sheet zero mode of the Liouville field to be identified with the target time on the *hypersurface* (36) of the extended space-time resulting after Liouville dressing.

We next remark that the restoration of the conformal invariance by the Liouville mode results in the following equations (c.f. (17)) for the σ -model background fields/couplings g^i near a fixed-point of the world-sheet renormalization group (large-times cosmology) we restrict ourselves here [22]:

$$(g^i)'' + Q(g^i)' = -\beta^i(g) , \quad (38)$$

where the prime denotes derivative with respect to the Liouville zero mode ϕ_0 , and the sign on the right-hand-side is appropriate for supercritical strings we are dealing with here. These equations replace the equilibrium equations $\beta^i = 0$ of critical string theory, and should be used in our colliding brane scenario to determine the evolution of the scale factor of the four-dimensional Robertson-Walker Universe. A preliminary analysis has been performed in [28], where we refer the reader for details.

Below we only describe briefly the main results. In our non-critical string scenario, one does indeed obtain an expanding Universe, in contrast to standard ekpyrotic scenaria [40, 41], based on critical strings and specific solutions to classical equations of motion. One of the most important features of the existence of a non-equilibrium phase of string theory due to the collision is the possibility for an *inflationary phase*. Although the physics near the collision is strongly coupled, and the σ -model perturbation theory is not reliable, nevertheless one can give compelling physical arguments favoring the existence of an early phase of the brane world where the four-dimensional Universe scale factor undergoes exponential growth (inflation). This can be understood as follows: in our model we encounter two type-II string theory branes colliding, and then bouncing back. From a stringy point of view the collision and bounce will be described by a phase where open strings stretch between the two branes worlds (which can be thought of as lying a few string scales apart during the collision, c.f. fig. 6). During that early phase the excitation energy of the brane worlds can be easily computed by the same methods as those used to study scattering of type II D-branes in [42]. In type II strings the exchange of pairs of open strings is described by annulus world-sheet diagrams, which in turn results in the appearance of “spin structure factors” in the scattering amplitude. The latter are expressed in terms of appropriate sums over Jacobi Θ functions. Due to special properties of these functions, the spin structures start of at quartic order in u [42]. The resulting excitation energy is therefore of order $\mathcal{O}(u^4)$ and may be thought off as an initial value of the central charge deficit of the non-critical string theory describing the physics of our brane world after the collision. The deficit

Q^2 is thus cut off at a finite value in the (world-sheet) infrared scale (early target times, c.f. fig. 6). One may plausibly assume that the central charge deficit remains constant for some time, which is the era of *inflation*. It can be shown [28] that for (finite) constant $Q^2 = Q_*^2 = \mathcal{O}(u^4)$ the Liouville equations imply a scale factor exponentially growing with the Liouville zero mode $a(\phi_0) = e^{|Q_*|\phi_0/2}$ (assuming a negative Q_*). Upon the condition (36), then, one obtains an early inflationary phase after the collision, in contrast to the critical-string based arguments of [41]. The duration of the inflationary phase is $t_{\text{inf}} \sim 1/|Q_*| \sim \mathcal{O}(u^{-2})$, which yields the conventional values of inflationary models of order $t_{\text{infl}} \sim 10^9 t_{\text{Planck}}$ for $u^2 \sim 10^{-9}$.

Exit from the inflationary phase is possible, and the de Sitter phase is assumed to be succeeded by a phase, to be identified with the *nucleosynthesis era*, in which the central charge deficit passes through a metastable critical phase in which Q vanishes, and then changes sign (cf. fig. 6). This can happen in Liouville strings with time-like fields X^0 , as we mentioned above, where the central charge deficit can oscillate before it reaches its equilibrium value [25, 27, 28]. During the metastable critical phase the central charge deficit is zero and hence there is no Liouville dressing. This implies that during that epoch any time dependence of fields on our brane world will be given by the ordinary critical string time dependence. In brane cosmologies the Friedman equation on the three brane, where matter lives, assumes the form [43] (for spatially flat branes):

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{|\Lambda_5|}{6} + \frac{\kappa^4}{36}\rho^2 + \frac{\mathcal{C}}{a^4} \quad (39)$$

where κ is the gravitational constant in the bulk geometry, \mathcal{C} is an integration constant (a mass of a bulk Schwarzschild black hole in the brane cosmology of [43], which for simplicity, and without any important physical consequences, we assume zero for our purposes), $\Lambda_5 < 0$ is the bulk anti de Sitter Cosmological constant, and a is the scale factor on the brane world. The density ρ denotes the total energy density on the brane, including matter contributions as well as cosmological constant (brane tension) contributions. Shifting ρ to make such tension contributions explicit, $\rho = c_0 + \rho_M$, where ρ_M is the matter density, one observes that for late times, the Friedman equation on the brane assumes the standard form, linear in ρ_M , *provided* one cancels the total vacuum energy as perceived by a brane (four-dimensional) observer: $c_0^2 - |\Lambda_5| = 0$. In our non-critical string case c_0 may be assumed positive, and one may identify $Q_0^2 = Q^2(\infty)$ (the equilibrium value of the string central charge deficit) with $c_0^2 - |\Lambda_5|$ which vanishes. During the nucleosynthesis era, therefore, the rate of expansion of the non-critical string Universe is that predicted by standard cosmology, which is a welcome feature given the phenomenological success of the nucleosynthesis model in explaining the abundance of light elements in the Universe.

At the end of the nucleosynthesis one faces a non-critical situation, with a central charge deficit significantly smaller than that of the inflationary era. The above considerations on the two times, Liouville ϕ and coordinate time X^0 , arise again. The generalized conformal invariance conditions (38) apply in this case, and hence the scale factor depends now on both X^0 and ϕ , but the evolution is confined for energetic reasons on the hypersurface (36), as explained above. We may assume that at the end of the nucleosynthesis $\beta^i = 0$ for the scale factor, and hence one obtains from (38) that a natural solution is

$$a(\phi, X^0) \sim a_0(X^0)\phi^{1/2-\mathcal{O}(u^2, H^2)}, \quad (40)$$

Since after the end of nucleosynthesis (present era) the universe is in a matter dominated phase, one may assume, from the above considerations, that $a_0 \sim (t)^{2/3}$ with $X^0 = -t$

the Robertson-Walker observable time to be connected with the Liouville time ϕ on the hypersurface (36). Thus the present era scale factor scales with time as: $a(t) \sim t^{2/3+1/2} = t^{7/6}$. This suffices to yield a current acceleration of the universe, thereby indicating that the dark energy (in the form of gravitational recoil-Liouville contributions) takes over the expansion. Unfortunately, such a scaling must not remain for ever, if one wants to recover an S-matrix equilibrium theory asymptotically (although it must be noted that this is not formally necessary in this framework, since Liouville strings do not admit S-matrix description in general, as we have discussed above).

Remarkably a solution to this problem can be provided in our model by noting the possibility of decompactification of the extra toroidal dimensions on the five branes, as a result of the collision. If one assumes a very slow decompactification (much slower than any other rate in the problem) then eventually the radius of the torus $R \rightarrow \infty$, while keeping the magnetic field energy $(HR)^2$ constant. The masses of the matter particles on the brane will therefore eventually go to zero, and hence one will enter a radiation era in which $\rho_M \sim 1/a^6$ on the five brane. From (39) this will imply an asymptotic scaling $a_0(t) \sim t^{1/3}$, which, upon Liouville dressing, will yield a scale factor $a \sim t^{1/3+1/2} \sim t^{5/6}$, as $t \rightarrow \infty$, thereby resolving the cosmic horizon problem (1), and thus saving the S-matrix in this model. This concludes our discussion on this toy model. It remains to be seen whether such conjectures/speculations can be accommodated in realistic string models of brane cosmologies.

Acknowledgements

I wish to thank Prof. H. Klapdor-Kleingrothaus and the organizers of the Conference *Beyond the Desert 2002*, held at Oulu, Finland, in June 2002, for their kind invitation and for providing a thought-stimulating atmosphere during the meeting.

References

- [1] For a recent review see: P. Biermann and G. Sigl, Lect. Notes Phys. **576** (2001) 1 [arXiv:astro-ph/0202425], and references therein
- [2] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration]: Nature **391**, 51 (1998); A. G. Riess *et al.* [Supernova Search Team Collaboration]: Astron. J. **116**, 1009 (1998); P. M. Garnavich *et al.*, Astrophys. J. **509**, 74 (1998).
- [3] J.R. Bond, A.H. Jaffe and L. Knox, Phys. Rev. D **57**, 2117 (1998); H. Lineweaver, astro-ph/9810334; N. A. Bahcall and X. Fan, astro-ph/9804082.
- [4] for a subset of references see: S. R. Coleman and S. L. Glashow, Phys. Rev. D **59** (1999) 116008 [arXiv:hep-ph/9812418]. S. R. Coleman and S. L. Glashow, arXiv:hep-ph/9808446. S. R. Coleman and S. L. Glashow, Phys. Lett. B **405** (1997) 249 [arXiv:hep-ph/9703240]; O. Bertolami, Gen. Rel. Grav. **34** (2002) 707 [arXiv:astro-ph/0012462]; Nucl. Phys. Proc. Suppl. **88** (2000) 49 [arXiv:gr-qc/0001097]. O. Bertolami and C. S. Carvalho, Phys. Rev. D **61** (2000) 103002 [arXiv:gr-qc/9912117].
- [5] T. Kifune, Astrophys. J. **518** (1999) L21 [arXiv:astro-ph/9904164]; G. Amelino-Camelia and T. Piran, Phys. Rev. D **64** (2001) 036005 [arXiv:astro-ph/0008107]. Phys. Rev. D **64** (2001) 036005 [arXiv:astro-ph/0008107]; Y. J. Ng, D. S. Lee, M. C. Oh and H. van Dam, Phys. Lett. B **507** (2001) 236 [arXiv:hep-ph/0010152]. J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Rev. D **63** (2001) 124025 [arXiv:hep-th/0012216].

- [6] S. Sarkar, *Mod. Phys. Lett. A* **17** (2002) 1025 [arXiv:gr-qc/0204092], and references therein.
- [7] K. Greisen, *Phys. Rev. Lett.* **16** (1966) 748; G. T. Zatsepin and V. A. Kuzmin, *JETP Lett.* **4** (1966) 78 [*Pisma Zh. Eksp. Teor. Fiz.* **4** (1966) 114].
- [8] T. Banks, W. Fischler: hep-th/0102077; S. Hellerman, N. Kaloper and L. Susskind, *JHEP* **0106** (2001) 003 [arXiv:hep-th/0104180]; W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, *JHEP* **0107** (2001) 003 [arXiv:hep-th/0104181]; E. Witten, hep-th/0106109; P. O. Mazur, E. Mottola: *Phys. Rev. D* **64**, 104022 (2001), and references therein.
- [9] R. J. Protheroe and H. Meyer, *Phys. Lett. B* **493** (2000) 1 [arXiv:astro-ph/0005349].
- [10] J. Ellis, N. E. Mavromatos, D. V. Nanopoulos: *Phys. Lett. B* **293** (1992) 37 [arXiv:hep-th/9207103]; *Mod. Phys. Lett. A* **10** (1995) 1685 [arXiv:hep-th/9503162]; for reviews see: *Erice Subnuclear Series* (World Sci., Singapore) **31** 1, (1993); [hep-th/9304133]; J. Chaos, Solitons and Fractals **10**, 345 (eds. C. Castro and M.S. El Naschie, Elsevier Science, Pergamon 1999) [hep-th/9805120], and references therein; G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *Int. J. Mod. Phys. A* **12** (1997) 607 [arXiv:hep-th/9605211]; J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *Phys. Rev. D* **62** (2000) 084019 [arXiv:gr-qc/0006004]. *Phys. Rev. D* **62** (2000) 084019 [arXiv:gr-qc/0006004]; E. Gravanis and N. E. Mavromatos, arXiv:hep-th/0103122.
- [11] L. Gonzalez-Mestres, arXiv:physics/0003080; L. J. Garay, *Phys. Rev. D* **58** (1998) 124015 [arXiv:gr-qc/9806047].
- [12] S. Chadha and H. B. Nielsen, *Nucl. Phys. B* **217** (1983) 125. H. B. Nielsen and I. Picek, *Nucl. Phys. B* **211** (1983) 269 [Addendum-ibid. *B* **242** (1984) 542]; J. L. Chkareuli, C. D. Froggatt and H. B. Nielsen, *Phys. Rev. Lett.* **87** (2001) 091601 [arXiv:hep-ph/0106036].
- [13] J. I. Latorre, P. Pascual and R. Tarrach, *Nucl. Phys. B* **437** (1995) 60 [arXiv:hep-th/9408016].
- [14] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, *Nature* **393** (1998) 763 [arXiv:astro-ph/9712103]; J. R. Ellis, K. Farakos, N. E. Mavromatos, V. A. Mitsou and D. V. Nanopoulos, *Astrophys. J.* **535** (2000) 139 [arXiv:astro-ph/9907340]; J. Ellis, N. E. Mavromatos, D. V. Nanopoulos and A. S. Sakharov, arXiv:astro-ph/0210124.
- [15] R. Gambini and J. Pullin, *Phys. Rev. D* **59** (1999) 124021 [arXiv:gr-qc/9809038]; arXiv:gr-qc/0110054.
- [16] A. Ashtekar and J. Lewandowski, *Class. Quant. Grav.* **14** (1997) A55 [arXiv:gr-qc/9602046]; A. Ashtekar, C. Rovelli and L. Smolin, *Phys. Rev. Lett.* **69** (1992) 237 [arXiv:hep-th/9203079]; C. Rovelli and L. Smolin, *Phys. Rev. Lett.* **61** (1988) 1155; *Nucl. Phys. B* **331** (1990) 80; For a comprehensive review (and more references) see: A. Ashtekar, arXiv:gr-qc/0112038.
- [17] S. D. Biller *et al.*, *Phys. Rev. Lett.* **83** (1999) 2108 [arXiv:gr-qc/9810044].
- [18] G. Amelino-Camelia, *Int. J. Mod. Phys. D* **11** (2002) 35 [arXiv:gr-qc/0012051]; *Nature* **418** (2002) 34 [arXiv:gr-qc/0207049]; G. Amelino-Camelia, D. Benedetti and F. D'Andrea, arXiv:hep-th/0201245; J. Kowalski-Glikman, *Phys. Lett. A* **286** (2001) 391 [arXiv:hep-th/0102098]; arXiv:hep-th/0209264.
- [19] J. Magueijo and L. Smolin, *Phys. Rev. Lett.* **88** (2002) 190403 [arXiv:hep-th/0112090]; arXiv:gr-qc/0207085.
- [20] M.B. Green, J.H. Schwarz, E. Witten: *Superstring Theory*, Vols I and II (Cambridge University Press, Cambridge 1987).

- [21] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995); *TASI lectures on D-branes*, hep-th/9611050; M. J. Duff, Sci. Am. **278**, 64 (1998).
- [22] F. David, Modern Physics Letters **A3** (1988) 1651; J. Distler, H. Kawai Nucl. Phys. **B321** (1989) 509; see also N.E. Mavromatos and J.L. Miramontes, Mod. Phys. Lett. **A4** (1989) 1847 (1989); E. D' Hoker and P.S. Kurzepa, Mod. Phys. Lett. **A5** (1990) 1411; E. D' Hoker, Mod. Phys. Lett. **A6** (1991) 745.
- [23] A.B. Zamolodchikov, JETP Lett. **43** (1986) 730.
- [24] D. Kutasov, N. Seiberg: Nucl. Phys. B **358** (1991) 600.
- [25] C. Schmidhuber, A. A. Tseytlin: Nucl. Phys. B **426** (1994) 187.
- [26] I. Antoniadis, C. Bachas, J. R. Ellis, D. V. Nanopoulos: Phys. Lett. B **211** (1988) 393; Nucl. Phys. B **328** (1989) 117; Phys. Lett. B **257** (1991) 278.
- [27] G. A. Diamandis, B. C. Georgalas, N. E. Mavromatos, E. Papantonopoulos and I. Pappa, Int. J. Mod. Phys. A **17** (2002) 2241 [arXiv:hep-th/0107124].
- [28] E. Gravanis and N. E. Mavromatos, arXiv:hep-th/0205298, Physics Lett. **B** in press, and work in progress.
- [29] The CTP formalism may be attributed to: J. Schwinger, J. Math. Phys. **2**, 407 (1961); for reviews see: E. Calzetta and B.L. Hu, Phys. Rev. **D37**, 2878 (1988); E. Calzetta, S. Habib and B.L. Hu, Phys. Rev. **D37**, 2901 (1988); H. Umezawa, *Advanced Field Theory: micro, macro and thermal concepts* (American Inst. of Physics, N.Y. 1993).
- [30] S. W. Hawking, Commun. Math. Phys. **87**, 395 (1982).
- [31] Y. I. Kogan, Phys. Lett. B **265** (1991) 269.
- [32] Y. I. Kogan, Univ. of British Columbia preprint UBCTP 91-13 (1991).
- [33] A non-linear difusive non-critical stringy system is examined in: N. E. Mavromatos and R. J. Szabo, Int. J. Mod. Phys. A **16** (2001) 209 [arXiv:hep-th/9909129]; see, however, how world-sheet supersymmetry may eliminate such non-linearities: N. E. Mavromatos and R. J. Szabo, JHEP **0110** (2001) 027 [arXiv:hep-th/0106259].
- [34] I. I. Kogan, N. E. Mavromatos and J. F. Wheeler, Phys. Lett. B **387** (1996) 483 [arXiv:hep-th/9606102]; N. E. Mavromatos and R. J. Szabo, Phys. Rev. D **59** (1999) 104018 [arXiv:hep-th/9808124].
- [35] J. Ellis, E. Gravanis, N. E. Mavromatos and D. V. Nanopoulos, arXiv:gr-qc/0209108.
- [36] D. Sudarsky, L. Urrutia and H. Vucetich, arXiv:gr-qc/0204027; J. Alfaro, H. A. Morales-Tecotl and L. F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000) [arXiv:gr-qc/9909079]; J. Alfaro and G. Palma, Phys. Rev. D **65**, 103516 (2002) [arXiv:hep-th/0111176].
- [37] For a review see: S. M. Carroll, Living Rev. Rel. **4**, 1 (2001) [astro-ph/0004075].
- [38] J. Ellis, N. E. Mavromatos, D. V. Nanopoulos: hep-th/0105206.
- [39] C. Bachas, arXiv:hep-th/9503030.
- [40] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **64**, 123522 (2001).
- [41] For a recent and concise review of the criticisms of the model of [40] see: A. Linde, arXiv:hep-th/0205259, and references therein.
- [42] C. Bachas, Phys. Lett. B **374**, 37 (1996) [arXiv:hep-th/9511043].
- [43] For a recent review see: D. Langlois, arXiv:hep-th/0209261 and references therein; see also: P. Binetruy, C. Deffayet and D. Langlois, Nucl. Phys. B **565** (2000) 269 [arXiv:hep-th/9905012]; P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B **477** (2000) 285 [arXiv:hep-th/9910219].